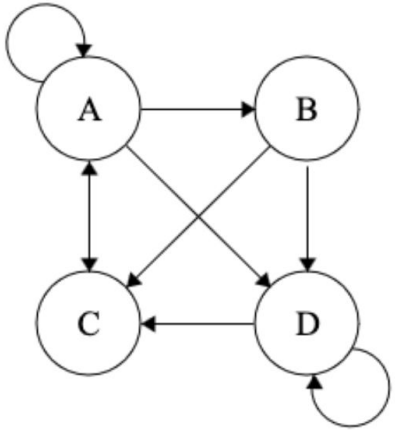
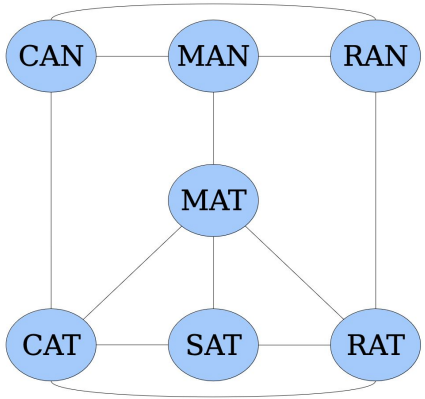
Graphs

Graphs represent **nodes** (or **vertices**) connected by **edges**.





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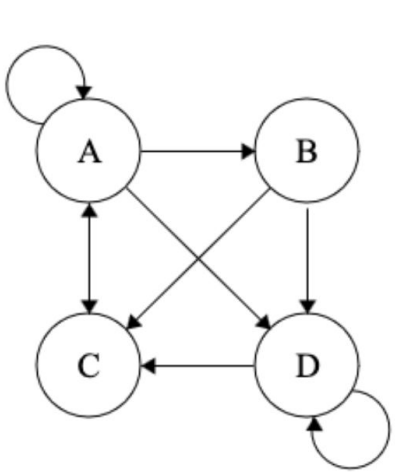
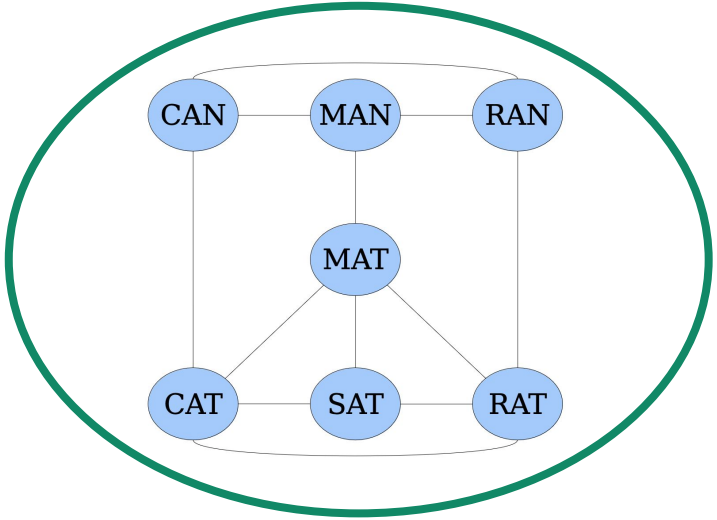




We will mostly be talking about **undirected graphs**, which have symmetric edges and don't allow self-loops.

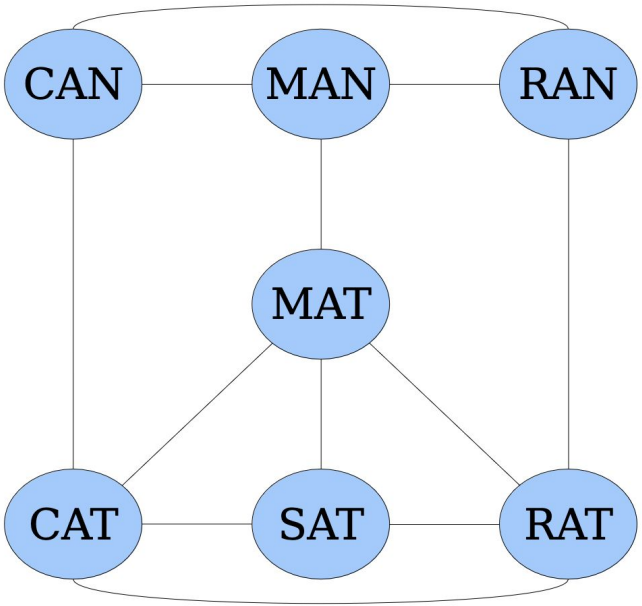
Graphs Concept Check

An undirected graph is formalized as **(V, E)** where:

- V is the set of all vertices
- E is the set of all edges
- An edge is an unordered pair of two vertices

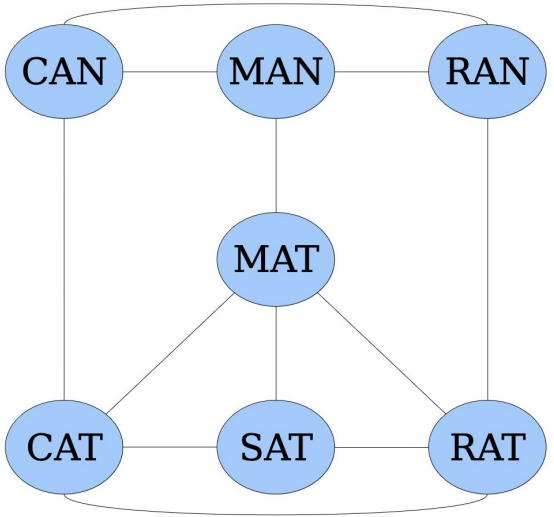
What does |V| mean?
What is |V| for this graph?

What does |E| mean?
What is |E| for this graph?



Traversing Graphs

- Two nodes u, v in a graph G = (V, E) are **adjacent** if {u, v} ∈ E
- **Walk**: a list of nodes where each node is adjacent to the next
  - N nodes → walk is length N-1
- Which of these are valid walks?
  - CAN, CAT, SAT, RAT, RAN, CAN
  - CAN
  - RAT, MAN



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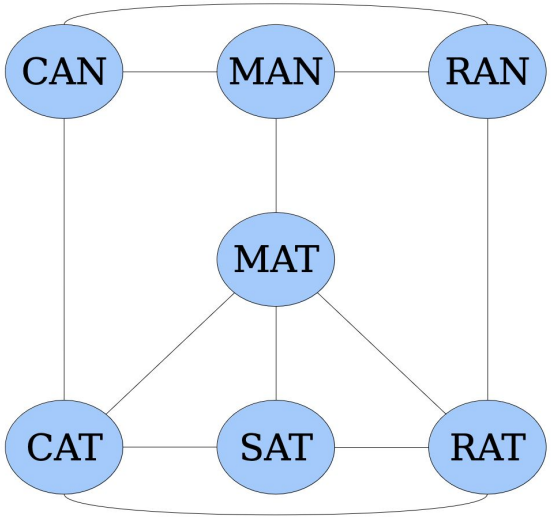
- Special types of walk!
  - **Closed Walk**: walk back to the same node you started with
  - e.g. CAN, CAT, SAT, RAT, RAN, CAN



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- Two nodes u, v in a graph G = (V, E) are **adjacent** if $\{u, v\} \in E$
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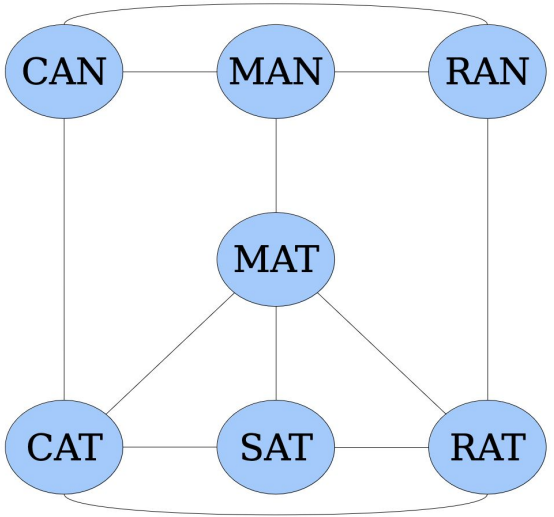
- Special types of walk!
  - **Closed Walk**: walk back to the same node you started with
  - **Path:** walk with no repeats
  - **Cycle**: a closed walk with no repeats of nodes or edges, except it goes back to the same node it started with (think “closed path”)

Traversing Graphs

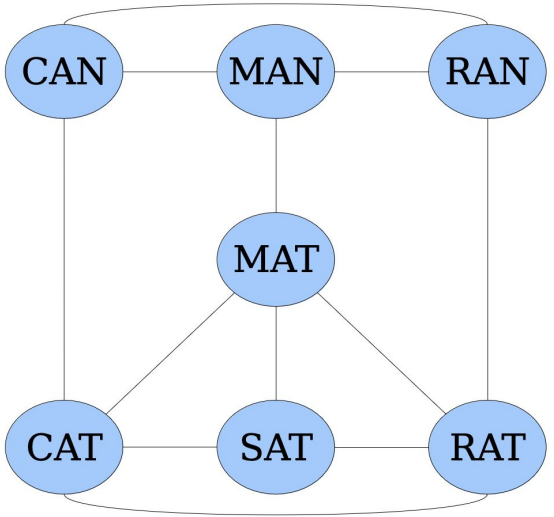
Can you come up with a length 7 cycle?



Graph Connectedness

- A node v is **reachable** from a node u if there is a path from u to v
- A graph is **connected** if any node is reachable from any other
- A **connected component** is a set consisting of a node and every node reachable from it

Is this graph connected?



Vertex Covers

A **vertex cover** C is a subset of nodes such that:

$$\forall x \in V.\ \forall y \in V.\ (\{x, y\} \in E \rightarrow (x \in C \vee y \in C))$$

("Every edge has at least one endpoint in C.")

Independent Sets

An **independent set** I is a subset of nodes such that

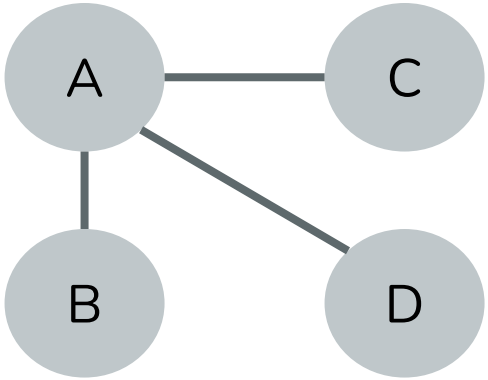
$$\forall u \in I.\ \forall v \in I.\ \{u, v\} \notin E.$$

("No two nodes in I are adjacent.")

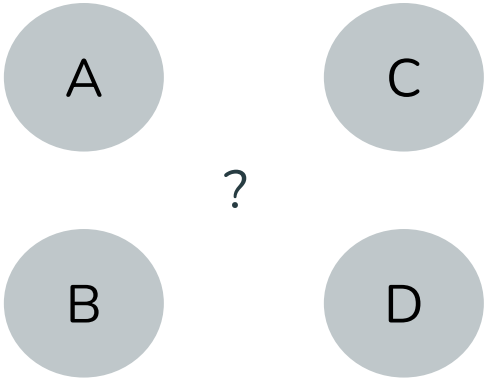
Complements

A graph $\mathbf{G}$'s **complement** $\mathbf{G^C}$ adds edges between any nodes that didn't have an edge in G, and removes all the original edges.

What's the complement of this graph?



$\mathbf{G}$



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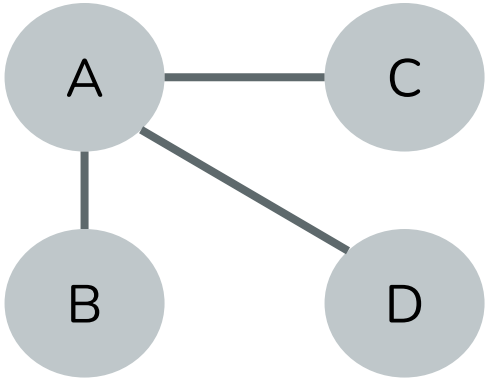
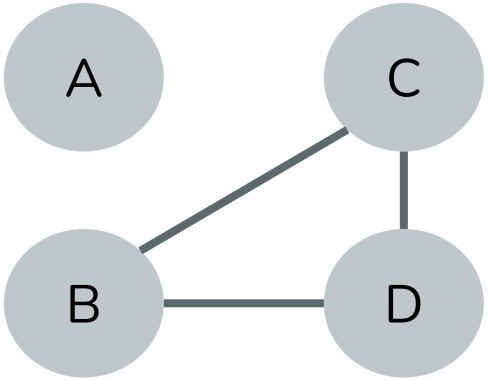


G



G^C

The Pigeonhole Principle:

If m objects are distributed into n bins **and m > n**,
then at least one bin will contain at least two objects.

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If m objects are distributed into n bins,
then some bin will have at least $\lceil m/n \rceil$ objects in it, and
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What does each principle say about [snack example]